Research Article

Determination of Wave Height and Wave Setup in the Surf Zone

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Abstract-**The coast of Bangladesh has certain feature in terms of significant beach profile. In this study, the wave height determine with linear beach profile from Dally's model has been discussed. The energy and momentum balance equations are solved by finite difference method. The result of wave height and wave set up shows good agreement with observed data.**

Keywords- **Water wave; wave height; wave setup; Surf Zone.**

I. INTRODUCTION

Wave breaking and energy dissipation is a complicated phenomenon in the shallow water region. When waves enter into shallow water region then its breaks and energy dissipates. But up to date, there are no models which explain the total energy dissipation properly. Several studies [e.g. Dally et al. (1984), Battjes and Janssen (1978), Hoque and Aoki (2002)] solved the water wave problems by numerical methods. Among them Dally et al. (1984) developed a model based on energy flux[4]. Battjes and Janssen (1978) proposed a model based on bottom friction[3]. Hoque and Aoki (2002) developed a model based on air bubble effects[6]. Although many works have been done based on different ideas, but in the transit zone no model works properly. The objective of this paper is to determine the wave height and wave setup and compare it with the experimental data conducted by Stive (1978).

II. ENERGY BALANCE

Energy cannot destroy. It can be transferred from one state to another state. When wave transformed into shallow water region then it breaks and energy dissipates. There is considered two sections named 1 and 2. The distance between two sections is Δx and energy flux at 1 is E_{f1} and at 2 is E_{f2}. The rate of change of energy flux at a distance Δx is proportional to the energy dissipation, D

$$\frac{E_{f2}-E_{f1}}{\Delta x} \propto -D$$

Now taking the limit $\Delta x \to 0$, then we have

$$\text{or,} \qquad \frac{dE_f}{dx} = -D \tag{1}$$

This is called energy balance equation. D is developed based on several assumptions. For example some models are given below:

Dally's model, $D = \frac{K}{h}\left[EC_g - EC_{gs}\right]$,

K is constant. Dally's (1984) developed this model based on energy flux variation[4].

Battjes and Janssen model (1978), $D = \frac{\alpha}{4T}\rho_w g \frac{H^3}{h}$,

α is constant. Battjes and Janssen (1978) developed energy dissipation in breaking solitary and periodic waves[3].

Hoque (2002), $D = 2\alpha\rho_w g V_a W_r$,

α is constant. Hoque (2002) developed energy dissipation model based on air bubbles effect in the surf zone[6].

III. MOMENTUM BALANCE

In the steady state, the shoreward flux of momentum must be independent of x, which we take perpendicular to the shore. Let us now consider the momentum in a slice of water bounded by the sloping surface $z = \bar{\eta}$, the sloping bottom $z = -h$ and two vertical planes $x = x_0$ and $x = x_0 + dx$. If the bottom slope is sufficiently small that $\overline{uw}$ and $\overline{u^2}$ at the bottom are negligible, then the flux of momentum into the slice, crossing the plane $x = x_0$ is

$$S_{xx} + \int_{-h}^{\bar{\eta}} P dz \tag{2}$$

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There is an additional flux of horizontal momentum due to the bottom pressure, since the bottom is not horizontal, amounting to

$$\rho g(\bar{\eta}+h)\frac{d\eta}{dx}dx$$

The validity of the approximations used here thus momentum balance gives

$$\frac{ds_{xx}}{dx}+\rho g(\bar{\eta}+h)\frac{d\eta}{dx}=0$$

$$\frac{d\bar{\eta}}{dx}=-\frac{1}{\rho g(\bar{\eta}+h)}\frac{dS_{xx}}{dx} \qquad (3)$$

IV. MODEL EQUATION

The model equation is

$$\frac{dE_f}{dx}=-D \qquad (4)$$

From Dally Model, $D=\frac{K}{h}\left[EC_g-EC_{gs}\right]$ where EC_g is now taken to be the depth integrated, time averaged energy flux as given by shallow water linear wave theory, and EC_{gs} is the energy flux associated with the stable wave that the breaking wave is striving to attain. We know energy flux, $E.C_g$, from equation (2) we may find

$$\frac{d(H^2\sqrt{h})}{dx}=-K\left[\frac{H^2}{\sqrt{h}}-(\gamma)^2\left(\sqrt{h}\right)^3\right]$$

Using forward difference method and taking step size Δx the discredited equation may be

$$\frac{(H^2\sqrt{h})_{i+1}-(H^2\sqrt{h})_i}{\Delta x}=$$

$$-\frac{K}{2}\left\{\left[\frac{H^2}{\sqrt{h}}-(\gamma)^2\left(\sqrt{h}\right)^3\right]_{i+1}-\left[\frac{H^2}{\sqrt{h}}-(\gamma)^2\left(\sqrt{h}\right)^3\right]_i\right\}$$

$$H_{i+1}{}^2=\frac{\frac{K\Delta x}{2}\left\{(\gamma)^2\left((h_{i+1})^{3/2}+(h_i)^{3/2}\right)-\frac{H_i{}^2}{\sqrt{h_i}}\right\}+H_i{}^2\sqrt{h_i}}{\sqrt{h}_{i+1}+\frac{K\Delta x}{2\sqrt{h}_{i+1}}}$$

The simplify form is

$$H_{i+1}=$$

$$\left\{\frac{K\Delta x\gamma^2\sqrt{h_ih_{i+1}}\left((h_{i+1})^{3/2}+(h_i)^{3/2}\right)+H_i^2\sqrt{h_{i+1}}(h_i-K\Delta x)}{\sqrt{h_i}(2\sqrt{h_{i+1}}+K\Delta x)}\right\}^{1/2} \qquad (5)$$

where K is dimensionless decay coefficient, γ is dimensionless coefficient whose value appears to lie somewhere between 0.35 to 0.40, and h_i, h_{i+1} and H_i are known quantity. we get the momentum balance equation,

$$\frac{d\bar{\eta}}{dx}=-\frac{1}{\rho g(h+\bar{\eta})}\frac{ds_{xx}}{dx}$$

Now applying finite difference method in differential term and average the other term, then

$$\frac{\bar{\eta}_{i+1}-\bar{\eta}_i}{\Delta x}=-\frac{2}{\rho g(h_{i+1}+h_i+\bar{\eta}_{i+1}+\bar{\eta}_i)}\left(\frac{S_{xxi+1}-S_{xxi}}{\Delta x}\right)$$

Finally we may get the discritization form of

Energy equation and momentum balance equation

as follows

$$H_{i+1}=$$

$$\left\{\frac{K\Delta x\gamma^2\sqrt{h_ih_{i+1}}\left((h_{i+1})^{3/2}+(h_i)^{3/2}\right)+H_i^2\sqrt{h_{i+1}}(h_i-K\Delta x)}{\sqrt{h_i}(2\sqrt{h_{i+1}}+K\Delta x)}\right\}^{1/2}$$

$$\bar{\eta}_{i+1}=\bar{\eta}_i-$$

$$\frac{2}{\rho g(h_{i+1}+h_i+\bar{\eta}_{i+1}+\bar{\eta}_i)}(S_{xxi+1}-S_{xxi})$$

V. RESULTS AND DISCUSSON

The comparison of wave height between model results and the data for *H_0/L_0=0.010*, *0.021*, *0.032* and *0.041* respectively. Figures 1, 2, 3 and 4 shows the graph of wave height for different $\frac{H_0}{L_0}=0.01$, 0.021, 0.032 and 0.041 respectively.

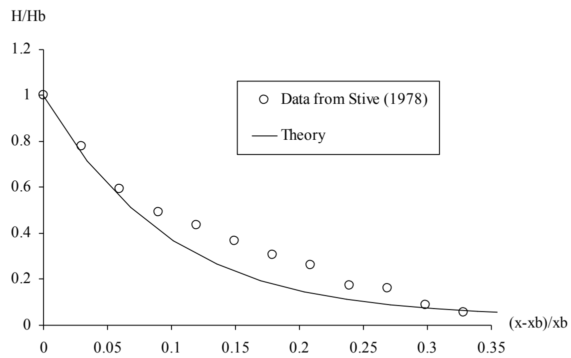

Figure 1. Comparison between theory and data of wave height ($\frac{H_0}{L_0}=0.01$)

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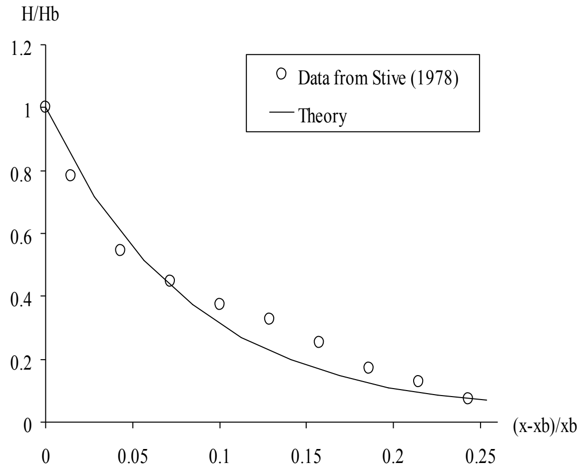

Figure 2. Comparison between theory and data of wave height ($\frac{H_0}{L_0}=0.021$ ***)***

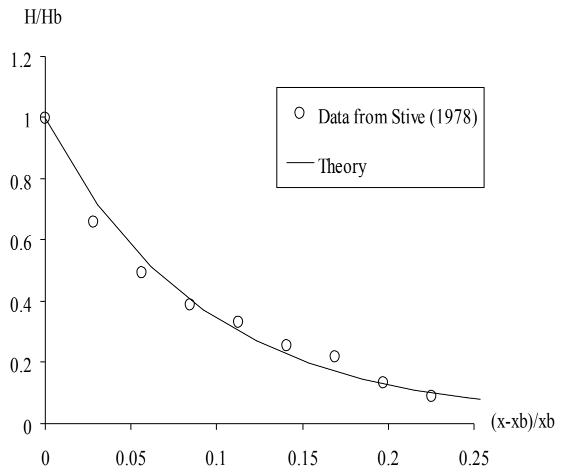

Figure 3. Comparison between theory and data of wave height ($\frac{H_0}{L_0}=0.032$ ***).***

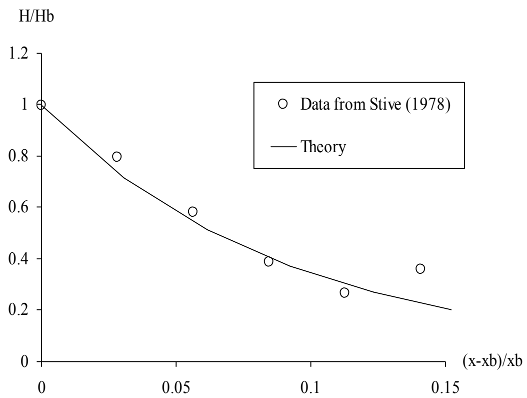

Figure 4. Comparison between theory and data of wave height ($\frac{H_0}{L_0}=0.041$ ***)***

There is discrepancy between theory and data around $0.1<\frac{x-x_b}{x_b}<0.3$. The computed wave height just after breaking point is decreasing with x. The scatter wave height is observed in the inner surf zone. The reason of this may be the caused of wave reflection from steeper beach. For the wave setup, from Figures (5), (6), (7) and (8) show the graph of wave setup for different $\frac{H_0}{L_0}$ = 0.010, 0.021, 0.032 and 0.041 respectively.

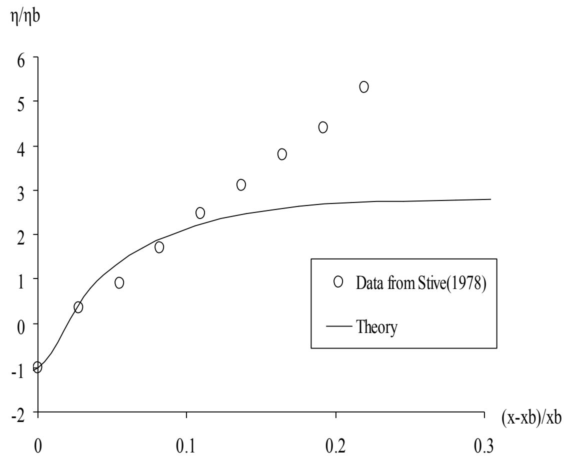

Figure 5. Comparison between theory and data of wave setup (H_0/L_0=0.010).

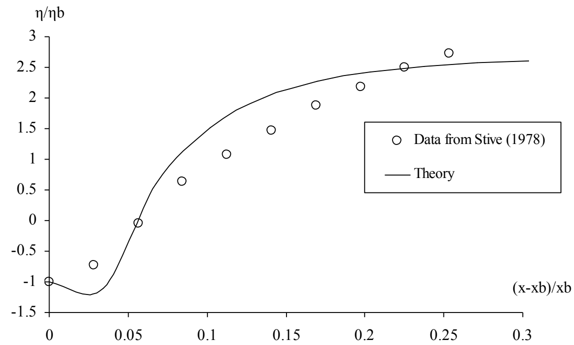

Figure 6. Comparison between theory and data of wave setup (H_0/L_0=0.021).

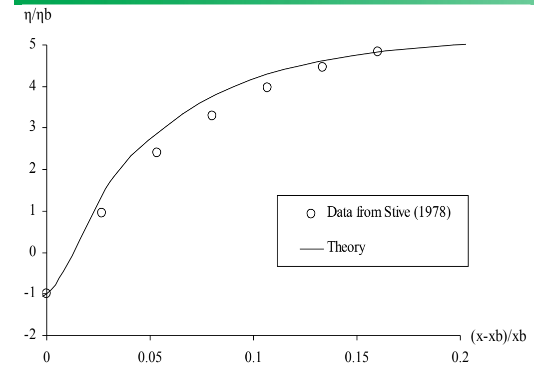

Figure 7: Comparison between theory and data of wave setup (H_0/L_0=0.032).

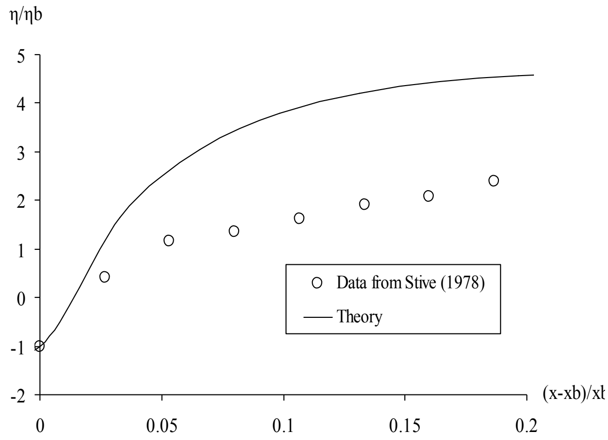

Figure 8. Comparison between theory and data of wave setup (H_0/L_0=0.041).

Figures (5), (6), (7) and (8) show the comparision of wave setup between model results and the data for H_0/L_0=*0.010*, *0.021*, *0.032* and *0.041* respectively. The agreements are well for the Figs. (6) and (7), but there are some inconsistence between theory and data in figures (5) and (8). In all the figures, computed wave setup is increasing with *x*. The inconsistence occurs for wave setup may be due to wave reflection from steeper beach and parameters. All of the figures show that after breaking point the measured data of wave setup keeps constants for some distance. However, the predicted setup curves do not follow the data for two apparent reasons. First, may not be good representation of the on shore excess momentum flux for near breaking and breaking conditions. Second, the measured data is nearly uniform for a distance after breaking is initiated. A similar variation can also be observed in other investigations such as Stive and Wind (1982)[11], Svendsen (1984)[12], Dally et al. (1984)[4] and Hoque and Aoki (2002)[6].

VI. CONCLUSION

The computed results for wave height and wave setup were verified by experimental data that was conducted by Stive (1978). The results showed good agreement between theory and data for wave height, but there was some inconsistence for wave setup. The reason could be occurred due to the wave reflection. Moreover, the data for wave setup kept constants some distances from the breaking point. It is interesting that even though the wave height is decreasing in this region, the momentum flux apparently not. Perhaps this because no energy is dissipated until the curl touches down and "white water" appears.

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